Studies on Non-linear Travelling Ionospheric Disturbances and Mode Coupling within the Auroral Region

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***Abstract*— In presence of non-linear perturbations of the thermospheric auroral region produced by Travelling Ionospheric Disturbances (TIDs) during the propagation of Atmospheric Gravity Waves (AGWs), an analytical expression of velocity of the constituent particles is derived through magnetohydrodynamic formalism. Using this, the expressions of Joule heating and viscous heating are derived, whose variations with time have been presented. The height-dependent ratio of the viscous heating rate to the Joule heating rate in generating AGW in the auroral electrojets is determined.**

Moreover, the mode coupling during transverse wave propagation within the auroral region is being explored, where the expressions of current and field during interactions are derived.

***Index Terms*— Auroral phenomena, ionospheric plasma, mode coupling, non-linear interaction, secondary wave field.**

I. INTRODUCTION

TRAVELLING Ionospheric Disturbances (TIDs), frequently observed at high and middle latitude, may be taken as the manifestations of ionospheric irregularities arising as the responses to Acoustic Gravity Waves (AGWs) [1]-[4].

The auroral region of the ionosphere may be characterized by different non-linear processes due to variations of the velocity distribution of the thermospheric constituents, medium temperature, ionizing frequency, effective collision frequency and recombination coefficient of electron and ions [5].

Auroral electric currents and charged particle precipitation initiate temperature enhancements and temperature fluctuations. The presence of fluctuating electric field introduces Joule heating along with viscous heating [6]. Magnetosphere-Ionosphere coupling mechanism also provides informations about Joule heating rate along with various other electrodynamic parameters. These heating make the largest contribution to the total energy budget in the medium.

Traveling Ionospheric Disturbances (TIDs) and atmospheric gravity waves (AGWs) has the origin at the auroral zone and these are associated with the auroral electrojet. The formation of AGWs in the auroral zone is dependent on Lorentz momentum forcing, Joule heating and heating due to particle precipitation.

The thermospheric constituent particles above 160 km altitudes is accelerated by the auroral electric field and in this region the Pedersen current predominates over the Hall current. The motion is collective in nature and associated with the movements of the constituent medium particles.

An analytical expression of the velocity of the upper thermospheric constituents in the auroral region has been derived through magnetohydrodynamic formalism. The expressions of Joule heating and viscous heating are derived. For different characteristic times, the variations of velocity with altitude are studied. The height-dependent ratio of the viscous heating to the Joule heating rate in generating AGWs in the auroral electrojets between 100 km – 150 km altitudes is derived. The results of statistical height-dependence of these two members in generating AGWs in the auroral electrojets have been compared with a previous work [7].

In the fields of radio-communication over long distances, the possibility of utilizing the secondary waves due to the coherent interaction of electromagnetic waves within the ionosphere has been explored. Here, such interaction within the ionosphere is considered. The regions of possible coupling between ordinary and extraordinary modes leading to the generation of ordinary mode will be presented.

II. MATHEMATICAL FORMULATIONS

The physical situation may be represented by the momentum balance equation, equation of continuity and equation of state as:

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$$mN\frac{\partial \vec{v}}{\partial t} + mN(\vec{v}\cdot\nabla)\vec{v} + mN\nu_e\vec{v} - mN\vec{g} = = -\nabla p - \vec{J}\times\vec{B} + \mu\nabla^2\vec{v} + \frac{1}{3}\mu\nabla(\nabla\cdot\vec{v}) \tag{1}$$

$$\frac{\partial N}{\partial t} + \nabla\cdot(N\vec{v}) = q - \alpha N^2 \tag{2}$$

$$p = Nk_B T \tag{3}$$

where N, the electron number density; ν_e, the effective electron-neutral molecule collision frequency; μ, the coefficient of viscosity of the medium; q, the rate of ionization; α, the electron-ion recombination coefficient and $\vec{B}$ is the geo-magnetic field vector.

The current density $\vec{J}$ may be expressed as

$$\vec{J} = (\sigma_0 - \sigma_1)(\vec{E}\cdot\hat{k})\hat{k} + \sigma_1(\vec{E} + \vec{v}\times\vec{B}) + + \sigma_2(\vec{E}\times\hat{k}) \tag{4}$$

where $\vec{E}$, the auroral electric field vector; σ_0, σ_1 and σ_2, the longitudinal, Pedersen and Hall conductivities respectively and the other symbols have their usual meaning. Fluctuations in number density and medium temperature are taken as

$N = N_0(1+\eta)$ and $T = T_0(1+\theta)$

η is the fluctuation in electron number density and

θ is the fluctuation in medium temperature.

Due to dominant nature of Pedersen conductivity over Hall conductivity in the region of interest (altitudes above about 160 km), Eq. (1) along with the perturbations can be expressed in a general form as

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} - a\frac{\partial v}{\partial t} - bv = c \tag{5}$$

The solution of Eq (5) is taken as the propagatory form:

$$v(x,z,t) = A\exp[\alpha_1(kx+pz-\omega t)] + + B\exp[\alpha_2(kx+pz-\omega t)] - \frac{c}{b} \tag{6}$$

where, α_1, α_2 are the roots of auxiliary equation.

The expressions of Joule heating (Q_J) and viscous heating (Q_V) can be deduced from:

$$Q_J = \sigma_1 E_x^2 + \sigma_1 E_y^2 + \sigma_0 E_z^2 + \sigma_1(B_z E_x - B_x E_z)v$$

$$Q_V = \mu v(k^2+p^2)\{A\alpha_1^2 \exp[\alpha_1(kx+pz-\omega t)] + + B\alpha_2^2 \exp[\alpha_2(kx+pz-\omega t)]$$

Its ratio, $R = \left|\frac{Q_V}{Q_J}\right|$

$$R = \frac{\begin{gathered}\mu(k^2+p^2)\{A\alpha_1^2\exp[\alpha_1(kx+pz-\omega t)] + \\ +\{B\alpha_2^2\exp[\alpha_2(kx+pz-\omega t)]\}\times \\ \times\{A\exp[\alpha_1(kx+pz-\omega t)] + \\ B\exp[\alpha_2(kx+pz-\omega t)] - \\ \frac{[k_B N_0 T_0\frac{\partial\theta}{\partial x} + \sigma_1 E_y B_z - \sigma_0 E_z B_y - mN_0(1+\eta)g]}{m\upsilon_e N_0(1+\eta) + \sigma_1 B_x B_y}\}\end{gathered}}{\begin{gathered}\sigma_1 E_x^2 + \sigma_1 E_y^2 + \sigma_0 E_z^2 + \sigma_1(B_z E_x - B_x E_z)\times \\ \times\{A\exp[\alpha_1(kx+pz-\omega t)] + B\exp[\alpha_2(kx+ \\ + pz-\omega t)] - \\ \frac{[k_B N_0 T_0\frac{\partial\theta}{\partial x} + \sigma_1 E_y B_z - \sigma_0 E_z B_y - mN_0(1+\eta)g]}{m\upsilon_e N_0(1+\eta) + \sigma_1 B_x B_y}\}\end{gathered}} \tag{7}$$

III. RESULTS AND DISCUSSION

The variations of average velocity with altitude for different characteristic times have been depicted in Fig. 1. The characteristic time is chosen as $\tau = \frac{mN}{\sigma_1 B^2}$ (the time constant required to reach the steady state). The inclusion of dotted lines is for functional completeness; these values are not physically valid because Hall conductivity is negligible in this region.

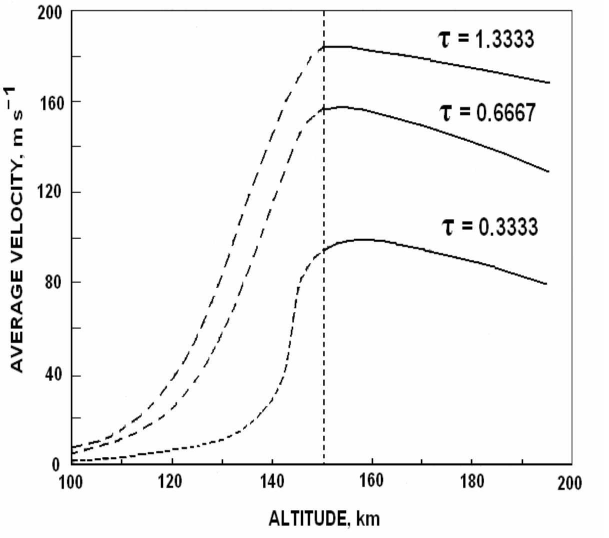

Figure 1. Average velocity vs. altitude at different characteristic times

In Figure 2, the heating rate has been plotted against the characteristic time. Joule heating is effective in the initial stage which decreases as the volume increases with time. When the motion is set-up due to dominating electric field, then only viscous heating results. This is increased due to movement and eventually it reaches a steady value.

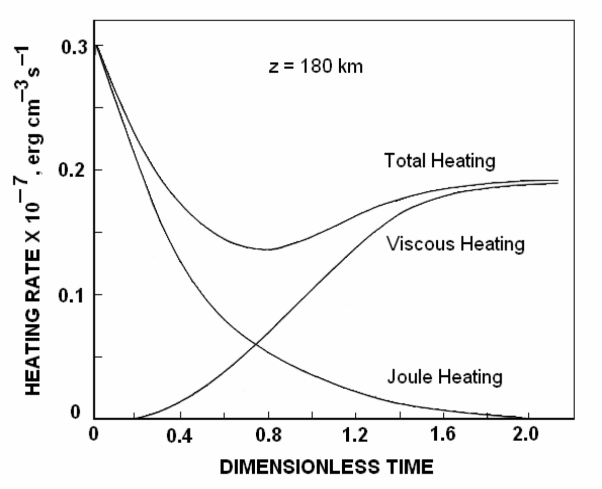

Figure 2. Heating rate vs. dimensionless time at z=180 km

Figure 3 shows the variations in the values for the ratio, *R*, derived from Eq. (7) with altitude ranging 100 to 150 km. The present work (continuous line) is compared with the previous work (dotted line) of Yuan *et al* [7].

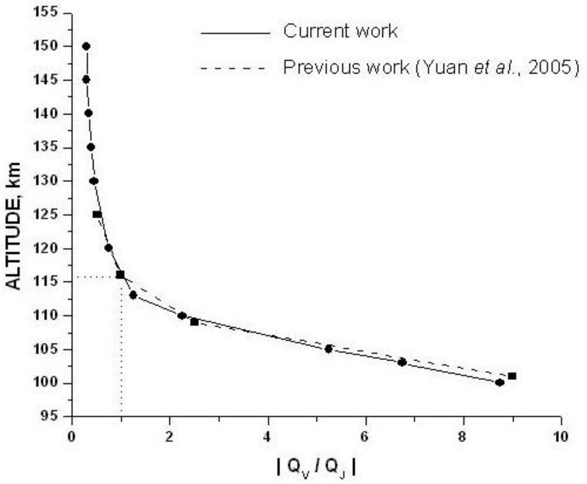

Figure 3. Variation of the ratio of viscous heating rate to Joule heating rate with altitude

IV. CONCLUSIONS

Joule dissipation of these currents is a major ionospheric energy source at auroral latitudes that seems to be several times greater than that directly associated with particle precipitation.

Auroral heating of the neutral atmosphere is a part of the more general question of the total energy budget. The propagation of gravity waves is generated mainly in the E-region of the ionosphere by Joule heating and Lorentz forcing.

Brekke pointed out that the relative importance of the two sources in generating the AGW is critically dependent on altitude [6]. The present work supports this prediction quite satisfactorily. Between 100 and 125 km altitude, the ratio ranges from 0.5 to 10 while between 125 and 150 km, it is approximately constant and the value obtained is 0.3. Under the height about 116 km the Lorentz force is relatively important and Joule heating dominates above the height.

Gravity waves propagating to thermospheric heights and interacting with the auroral ionospheric plasma give rise to TIDs. These can be registered by a riometer.

MODE COUPLING

In the presence of irregularities, the fluctuations are introduced by the method of stretching. The second-order equations may be written as

$$\left.\begin{aligned}\nabla\times\vec{H}_2 &= \frac{1}{c}\frac{\partial}{\partial t}[(\varepsilon_0)\vec{E}_2+(\varepsilon_1)\vec{E}_1]\\ \nabla\times\vec{E}_2 &= \frac{i\omega}{c}\vec{H}_2\end{aligned}\right\}\tag{8}$$

From these, one can get

$$[\nabla\nabla.-\nabla^2-\frac{\omega^2}{c^2}(\varepsilon_0)]\vec{E}_2=\frac{2\omega^2}{c^2}(\varepsilon_1)\vec{E}_1\tag{9}$$

Where, (ε_0) and (ε_1) are the zeroth order and first order matrices of (ε) derived from the appropriate form of Lorentz force equation consistent with the nature of the medium.

(ε) may be expressed as:

$$(\varepsilon)=I+\frac{i\omega_p^2}{\omega\chi(\eta-i2\omega)}\times\begin{pmatrix}\chi_x & \Omega_x\Omega_y+\Omega_z(\eta-i2\omega) & \Omega_z\Omega_x-\Omega_y(\eta-i2\omega)\\ \Omega_x\Omega_y-\Omega_z(\eta-i2\omega) & \chi_y & \Omega_y\Omega_z+\Omega_x(\eta-i2\omega)\\ \Omega_z\Omega_x+\Omega_y(\eta-i2\omega) & \Omega_z\Omega_y-\Omega_x(\eta-i2\omega) & \chi_z\end{pmatrix}$$

Where, $\Omega=\frac{eH}{mc}$, $\omega_p^2=\frac{4\pi ne^2}{m}$, $\chi=(\eta-i2\omega)^2+\Omega^2$.

For ordinary and extra-ordinary waves, it is considered:

$$E_1^{(0)}=\hat{z}\xi_{\|}e^{i\varphi}.$$

$$H_1^{(0)}=\hat{x}k_y\frac{c}{\omega}\xi_{\|}e^{i\varphi}$$

$$E_1^{(e)}=A(a\hat{x}+\hat{y})e^{i(\vec{k}\cdot\vec{r}-\omega t)}=\xi_e e^{i(kz-\omega t)}$$

$$H_1^{(e)}=-\hat{z}k_y\cdot\frac{cAa}{\omega}e^{i\varphi}$$

Where

$$k=\hat{y}k_y+\hat{z}k_z$$

$$\varphi=k_y y-\omega t$$

$$A^2=\frac{|\xi_e|^2}{|a|^2+1},\quad a=\frac{i\omega_{p0}^2\Omega_{0z}}{\omega\chi_0+i\omega_{p0}^2(\eta-i2\omega)}$$

A= the normalization factor.

$$\omega_{p0}^2=\frac{4\pi n_0 e^2}{m},\quad \Omega_{0z}=\frac{eH_{0z}}{mc}$$

$$\chi_0=(\eta-i2\omega)^2+\Omega_0^2.$$

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Eq. (9) contributes to the process of coupling through the part $\mathrm{Re}\frac{2\omega^2}{c^2}(\varepsilon_1)\vec{E}_1 = \vec{J}_s$

The expression of second-order current and field during interaction between ordinary (ω_1) and extra-ordinary modes (ω_2) of propagation are obtained as:

$$\begin{aligned}\vec{J}_S = \hat{z}\frac{\omega_\pm^2}{c^2}Ae^{-(K_1^{''}+k_2^{''})y}[\{-\xi_1 a'(\zeta_1 k_2^{''}+\zeta_2 k_2^{'})\mp \\ \mp\xi_1 a''(\zeta_1 k_2^{'}-\zeta_2 k_2^{''})+\xi_2(\zeta_3 k_2^{''}+\zeta_4 k_1^{'})\} \\ \cos(k_\pm y-\omega_\pm t)+\{\mp\xi_1 a'(\zeta_1 k_2^{'}-\zeta_2 k_2^{''})+ \\ +\xi_1 a_2^{'}(\zeta_1 k_2^{''}+\zeta_2 k_2^{'})\pm \\ \pm\xi_2(\zeta_3 k_2^{'}-\zeta_4 k_2^{''})\}\sin(k_\pm y-\omega_\pm t)],\end{aligned} \quad (10)$$

and,

$$\vec{E}_2 = \vec{J}_S[k_\pm^2 - \frac{\omega_\pm^2}{c^2}(1-\frac{2\omega_{P0}^2}{\eta^2+4\omega^2})]^{-1} \quad (11)$$

where symbols have their own meaning.

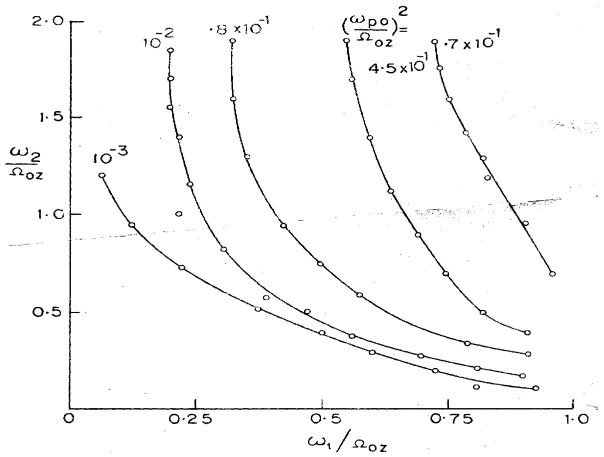

Figure 4. Interaction of ordinary wave (ω_1) with extraordinary wave (ω_2) leading to the generation of ordinary wave ($\omega_1 - \omega_2$).

For different value of plasma frequency (ω_p), the value of ω_1 and ω_2 are varied in step. For the value of ω_p, there is a curve in the $(\omega_1 - \omega_2)$ plane which characterizes the allowable regions of interaction between ordinary and extra-ordinary modes of propagation (Fig. 4), where the regions of possible coupling between the modes, leading to the generation of ordinary mode $\omega_1 \pm \omega_2$, have been shown. The coupling between ordinary and extra-ordinary modes can be important under quasi-transverse condition.

Coupling phenomena at various heights of the ionospheric medium have been investigated by different investigators. EISCAT observations are being used successfully to investigate the dynamical coupling of the auroral F-region ionosphere and thermosphere [8], [9].

High power and high frequency heating experiments from different locations of the world have been used to explore the electrical processes of the lower and upper atmosphere along with various possible sources of energy transfer between the near and far above the earth surface. Transport of electromagnetic energy from the atmosphere to the magnetosphere via ionosphere and back to the earth surface via the ionosphere and lower atmosphere through various coupling mechanism is a subject of current interest [10], [11]. Through the present analysis, an attempt has been taken which is enable to study a physical process within the disturbed ionosphere.

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