

Studies on the Short-range pressure pulse generation within the auroral Ionosphere

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Abstract

Auroral electric field is a probable source of Atmospheric Gravity Waves (AGW) in the thermospheric region of Auroral zone of the ionosphere and associated Traveling Ionospheric Disturbances (TID). This field is considered to be active through Lorentz force and Joule dissipation that influence the neutral gas of the medium through collision. The process introduces the short-range gravity waves. The expressions for the low frequency part of the fractional pressure variations have been derived within the auroral region of the ionosphere. The results of numerical analyses are presented graphically.

1. Introduction

Traveling Ionospheric Disturbances (TIDs) are manifestations of ionospheric irregularities arising as response to Atmospheric Gravity Waves (AGW). Due to the passage of the AGW, the ions being forced along the field lines by the neutral air winds driven by the pressure wave. The auroral electric field is mainly responsible for TIDs and partly for AGWs in the thermospheric region [1, 2]. Different investigations are made to explore the variability of observed gravity waves and its relationship with the TIDs, so that the upward energy flow from troposphere to mesosphere via stratosphere can be understood. This way of energy coupling from lower to upper atmosphere along with the resulting ionospheric effects gives the overall upper atmospheric energy balance [3].

Auroral region of the ionosphere may be characterized by different non-linear processes due to variations of the velocity distribution of the thermospheric constituents, medium temperature, ionizing frequency, effective collision frequency and recombination coefficient of electron and ions [4].

One of the important energy sources in the auroral ionosphere is Joule heating and this arises from the electric currents generated by geomagnetic disturbances. These cause generation of gravitational waves that propagate both horizontally and vertically in the upper atmosphere. During magnetic disturbances, a significant increase in thermospheric temperature and associated changes in the composition of the medium occur.

The variations in the nighttime ionospheric parameters during the passing of AGWs over the site of observations are mainly caused by the redistribution of the plasma ions along the magnetic field lines [5], due to which there will be uneven perturbations in the formation and distributions of charged particles that set up peculiarities in TIDs.

Using magnetohydrodynamic formalism, a model calculation is performed through which the magnitude and the form of the anticipated atmospheric wave-train are obtained [6]. Auroral electric field under the stated circumstances is considered to be active through Lorentz force and Joule dissipation via the neutral gas of the medium thereby introducing short-range gravity waves. The expressions for the low-frequency part of the fractional pressure variations have been derived for the auroral region of the ionosphere which appears as pressure pulses. The results of numerical analyses are presented graphically.

2. Mathematical formulations

The physical situation may be represented by the following momentum balance equation, equation of continuity, equation of state and heat balance equation

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) - \rho \vec{g} + \nabla p = -\vec{J} \times \vec{B} + \mu \nabla^2 \vec{v} + \frac{1}{3} \mu \nabla (\nabla \cdot \vec{v}) - \rho \nu \vec{v} \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = q - \alpha \rho^2, \quad p = \rho k_B T \quad (2)$$

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) (\rho \rho^{-\gamma}) = (\gamma - 1) \rho^{1-\gamma} S \quad (3)$$

along with the Maxwell's field equations.

Current density in this model of the auroral system has been taken as

$$\vec{J}(x, z, t) = \hat{j} A T(t) \exp[-\beta_1^2 (x - x_0)^2 - \beta_2^2 (z - z_0)^2] \quad (4)$$

$\beta_1^{-1}, \beta_2^{-1}$ are the cross-section parameters, Total current is represented by $\frac{\pi A}{\beta_1 \beta_2}$.

In the presence of perturbations, introducing various transformations, the expressions of Joule heating (Q_J) and viscous heating (Q_V) have been derived as:

$$Q_J = \sigma_1 E_x^2 + \sigma_1 E_y^2 + \sigma_0 E_z^2 + \sigma_1 (B_z E_x - B_x E_z) \times \\ \times [C_1 \exp\{\xi_1 (kx + pz - \omega \delta \tau)\} + C_2 \exp\{\xi_2 (kx + pz - \omega \delta \tau)\} - \frac{c}{b}] \quad (5)$$

$$Q_V = \mu \nu (k^2 + p^2) [C_1 \xi_1^2 \exp\{\alpha_1 (kx + pz - \omega \delta \tau)\} + C_2 \xi_2^2 \exp\{\alpha_2 (kx + pz - \omega \delta \tau)\}] \quad (6)$$

Where, $\tau = \frac{\rho}{\sigma_1 B^2}$, the characteristic time and $\delta = \frac{t}{\tau}$ is the dimensionless time.

Eliminating ρ and ν , one can get,

$$\nabla^2 p + (b_1 - ib_2) \frac{\partial^2 p}{\partial z^2} + (b_3 - ib_4) \frac{\partial p}{\partial z} + (b_5 + ib_6) p = M_1(x, z) \quad (7)$$

Where,

$$M_1(x, z) \equiv (d_1 - id_2) F_z - d_3 \frac{\partial^2 S}{\partial z^2} + (d_4 - id_5) \quad (8)$$

Different substitutions are made here. Within these, there are terms representing Brunt-Vaisala frequency, i. e., the internal gravity wave cut-off frequency $\omega_g = \sqrt{\frac{(\gamma-1)g}{\gamma H}}$ and acoustic cut-off frequency $\omega_A = \frac{1}{2} \sqrt{\frac{2g}{H}}$.

With the transformation, $p = e^{\frac{z}{2H}} \phi(x, z)$, (9)

equation (7) takes the form $\left(\frac{\partial^2}{\partial x^2} + A_1 \frac{\partial^2}{\partial z^2} + A_2 \right) \phi(x, z) = M_2$ (10)

Where,

$$M_2 = e^{-\frac{z}{2H}} M_1 \\ A_1 = (1 + b_1) - ib_2 \\ A_2 = \frac{1 + b_1 + 2Hb_3 + 4H^2b_5}{4H^2} - i \frac{b_2 + 2Hb_4 - 4H^2b_6}{4H^2}, \text{ with} \\ \frac{1}{H} [(1 + b_1 + Hb_3) - i(b_2 + Hb_4)] = 0.$$

Further transformation, $t = \left[A_2 \left(x^2 + \frac{z^2}{A_1} \right) \right]^{\frac{1}{2}}$, (11)

on the homogeneous equation from (10) yields Bessel's equation of order zero

$$\left(\frac{d^2}{dt^2} + \frac{1}{t} \frac{d}{dt} + 1 \right) \phi = 0$$

The solution may be written in the form of Green's function as

$$\phi^G(x, z : x', z') = \frac{i}{4\sqrt{A_1}} H_0^2 \left[A_2 \left\{ (x-x')^2 + \frac{(z-z')^2}{A_1} \right\} \right]^{\frac{1}{2}}$$

Where H_0^2 is Hankel's Bessel function of the third kind.

$$\text{Thus, } \phi(x, z) = \frac{i}{4\sqrt{A_1}} \iint dx' dz' M_2(x', z') H_0^2 \left[A_2 \left\{ (x-x')^2 + \frac{(z-z')^2}{A_1} \right\} \right]^{\frac{1}{2}}$$

Taking Fourier transformation of (10) and using (9), one can get

$$p(x, z, t) = \frac{ie^{\frac{z}{2H}}}{8\pi\sqrt{A_1}} \int_{-\infty}^{+\infty} d\omega \iint dx' dz' M_2(x', z') e^{i\omega x} H_0^2 \left[A_2 \left\{ (x-x')^2 + \frac{(z-z')^2}{A_1} \right\} \right]^{\frac{1}{2}} \quad (12)$$

Equation (12) can be simplified as

$$p_L(x, z, t) = -i \frac{e^{\frac{z}{2H}}}{2\pi} \iint dx' dz' \int_0^{+\infty} d\tau \frac{N_L \left\{ t - (t_L^2 + \tau^2)^{\frac{1}{2}} \right\} e^{-\frac{z}{2H}} \cos[b_4(t^2 - t_L^2)^{\frac{1}{2}}]}{(t^2 - t_L^2)}$$

where $\tau^2 = t^2 - t_L^2$

$$\therefore p_L(x, z, t) = -i \frac{e^{\frac{z}{2H}}}{2\pi} \iint dx' dz' \int_0^{+\infty} \frac{d\tau}{\tau} N_L \left\{ t - (t_L^2 + \tau^2)^{\frac{1}{2}} \right\} e^{-\frac{z}{2H}} \cos(b_4 \tau) \quad (13)$$

This implies the low frequency component of the fractional pressure field generated by an extended line source of heating in the atmosphere. Now, the Joule heating source term has been chosen exactly as the current density source term:

$$S(x, z, t) = DT(t) \exp \left[\frac{\left\{ -\beta_1^2 (x-x_0)^2 - \beta_2^2 (z-z_0)^2 \right\}}{\rho_0(z)} \right] \quad (14)$$

Thus, the low frequency parts of the fractional pressure variations are:

$$p_L(x, z, t) = -\frac{7D}{4g^2 H^{3/2} \rho_0(\bar{z}_0) \beta_1 \beta_2 t_L} \exp \left[-\left(\frac{1}{4H\beta_2} \right)^2 + \frac{(z-\bar{z}_0)}{2H} \right] \left(\frac{b_2}{b_1} \right)^{1/2} \phi_J(x, z, t) \quad (15)$$

$$p_L(x, z, t) = -\frac{A(x-\bar{x}_0)B_0}{2gH^{3/2} \rho_0(\bar{z}_0) \beta_1 \beta_2 R^2} \exp \left[-\left(\frac{1}{4H\beta_2} \right)^2 + \frac{(z-\bar{z}_0)}{2H} \right] \left(\frac{b_2}{b_1} \right)^{1/2} \phi_L(x, z, t) \quad (16)$$

Where,

$$\phi_{J,L} = \int_0^{+\infty} \frac{d\tau}{\tau} N_L \left\{ t - (t_L^2 + \tau^2)^{1/2} \right\} e^{-\frac{z}{2H}} \cos(b_4 \tau) \quad (17)$$

$$N_L(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega e^{i\omega x} d_3 \frac{\partial^2 S}{\partial z^2} M_L(x', z') \text{ for Joule heating} \quad (18)$$

$$N_L(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega e^{i\omega x} d_1 F_z \text{ for Lorentz heating} \quad (19)$$

Various physical parameters for the auroral current systems have been chosen properly. $H=10 \text{ km}=10^4 \text{ m}$; $\gamma=1.4$; $C_L = \frac{R}{t_L} = 350 \text{ ms}^{-1}$; $g=9.5 \text{ ms}^{-2}$; $B_0 = 6 \times 10^{-5} \text{ Wm}^{-2}$; $\rho_0(\bar{z}_0) = 2.4 \times 10^{-8} \text{ kg m}^{-3}$; $\frac{\pi A}{\beta_1 \beta_2} = 10^5 \text{ A}$;

Computations for the low frequency part of the fractional pressure variations are performed by the use of FORTRAN subroutine programs.

3. Discussion

The nature of pressure pulse variations in altitude time domain are shown in Figure 1. The loci of pressure maxima due to Joule heating have been plotted in the upper panel, while the lower panel depicts the loci of pressure maxima due to Lorentz heating. The plots are drawn for a given horizontal range $(x-\bar{x}_0) = 500 \text{ km}$.

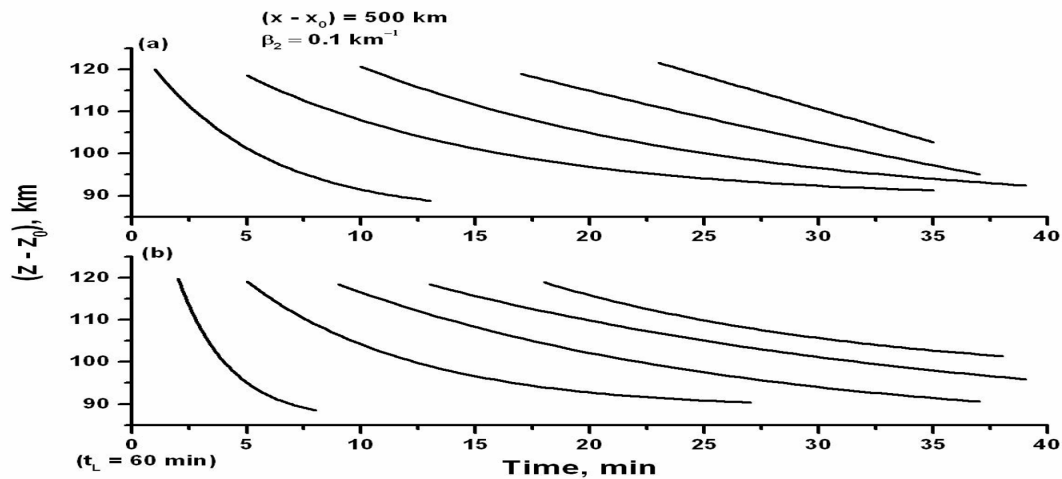


Figure 1. Loci of pressure maxima produced due to Joule heating [Upper panel (a)] and Lorentz forcing [Lower panel (b)] at a point 500 km away from the source point

In the auroral region, the study of the effects of charged particles on the neutral atmospheric processes have also been executed through the parameterization of ion drag and joule heating in the momentum balance and energy balance equations respectively, where the influence of different conductivities in the medium are taken into account [7].

The role of gravity waves in the production of variability of atmospheric tide, observed through Upper Atmospheric Research Satellite (UARS), has not been explored sufficiently. The generations of pressure pulses by the AGWs, the influence of tide modulating the AGWs and orographic forcing as a source of mesospheric gravity waves in the process of the generation of TIDs have not yet been sufficiently explored. Further works in some of these areas are contemplated.

4. References

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