

Studies on non-linear Travelling Ionospheric Disturbances and mode coupling within the auroral region

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Outline of the work

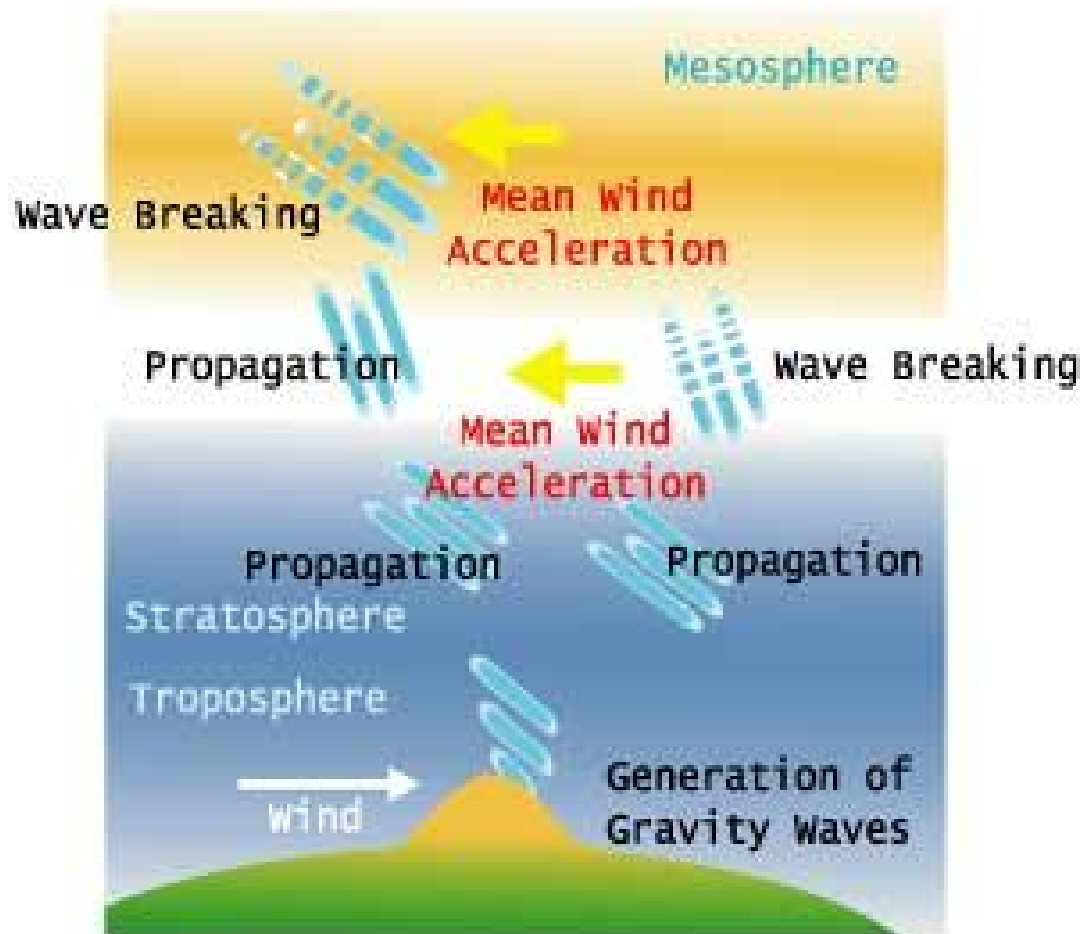
- Analytical expression of velocity of the constituent particles of auroral region is derived through magnetohydrodynamic formalism
- Expressions of Joule heating and viscous heating are derived, variations with time have been presented
- Height-dependent ratio of the viscous heating rate to the Joule heating rate in generating AGW in the auroral electrojets is determined
- mode coupling during transverse wave propagation within the auroral region is being explored, the expressions of current and field during interactions are derived

Atmospheric Gravity Waves (AGW) are the oscillations of air parcels by the lifting force of bouyancy and the restoring force of gravity. These waves propagate vertically as well as horizontally, and actively transport energy and momentum from the troposphere to the middle and upper atmosphere.

These are caused by a variety of sources

- including the passage of wind across terrestrial landforms
- interaction at the velocity shear of the polar jet stream and
- radiation incident from space.

AGWs can be seen in the higher atmosphere (100-200km) by moving bands of atmospheric air glow. The airglow emits spectra from chemiluminescence of atmospheric molecules.



Schematic of the formation of AGWs and its propagation

Travelling Ionospheric Disturbances (TIDs) may be taken as the manifestations of ionospheric irregularities arising as the responses to **Atmospheric Gravity Waves (AGWs)**

TIDs are frequently observed at high and middle latitude

Traveling Ionospheric Disturbances (TIDs) and atmospheric gravity waves (AGWs) has the origin at the auroral region and these are associated with the auroral electrojet

Different non-linear processes involved in auroral region of the ionosphere

These are mainly due to variations of

- the velocity distribution of the thermospheric constituents
- medium temperature
- ionizing frequency
- effective collision frequency and
- recombination coefficient of electron and ions

The formation of AGWs in the auroral region is dependent on

- Joule heating
- Lorentz momentum forcing and
- heating due to particle precipitation.

The presence of fluctuating electric field introduces Joule heating along with viscous heating. Magnetosphere-Ionosphere coupling mechanism also provides informations about Joule heating rate along with various other electrodynamic parameters. These heating make the largest contribution to the total energy budget in the medium.

Auroral electric currents and charged particle precipitation initiate temperature enhancements and temperature fluctuations.

The thermospheric constituent particles above 160 km altitudes are accelerated by the auroral electric field and in this region the Pedersen current predominates over the Hall current. The motion is collective in nature and associated with the movements of the constituent medium particles.

MATHEMATICAL FORMULATIONS

- momentum balance equation

$$\begin{aligned} mN \frac{\partial \vec{v}}{\partial t} + mN (\vec{v} \cdot \nabla) \vec{v} + mN \nu_e \vec{v} - mN \vec{g} = \\ = -\nabla p - \vec{J} \times \vec{B} + \mu \nabla^2 \vec{v} + \frac{1}{3} \mu \nabla (\nabla \cdot \vec{v}) \end{aligned}$$

- equation of continuity

$$\frac{\partial N}{\partial t} + \nabla \cdot (N \vec{v}) = q - \alpha N^2$$

- equation of state

$$p = Nk_B T$$

- N , the electron number density
- ν_e , the effective electron-neutral molecule collision frequency
- μ , the coefficient of viscosity of the medium
- q , the rate of ionization
- α , the electron-ion recombination coefficient
- \vec{B} , the geo-magnetic field vector

The current density \vec{J} may be expressed as

$$\vec{J} = (\sigma_0 - \sigma_1)(\vec{E} \cdot \hat{k})\hat{k} + \sigma_1(\vec{E} + \vec{v} \times \vec{B}) + \sigma_2(\vec{E} \times \hat{k})$$

- \vec{E} , Auroral electric field vector
- σ_0 , Longitudinal conductivity
- σ_1 , Pedersen conductivity
- σ_2 , Hall conductivity

other symbols have their usual meanings.

Fluctuations in number density and medium temperature are taken as

$$N = N_0(1 + \eta) \quad \text{and} \quad T = T_0(1 + \theta)$$

- η , the fluctuation in electron number density
- θ , the fluctuation in medium temperature

Momentum balance equation along with the perturbations can be expressed in a general form as

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} - a \frac{\partial v}{\partial t} - bv = c$$

Solution in propagatory form

$$v(x, z, t) = A \exp[\alpha_1 (kx + pz - \omega t)] + B \exp[\alpha_2 (kx + pz - \omega t)] - \frac{c}{b}$$

where

$$a = \frac{mN_0(1 + \eta)}{\mu}$$

$$b = \frac{m v_e N_0 (1 + \eta) + \sigma_1 B_x B_y}{\mu}$$

$$c = \frac{k_B N_0 T_0}{\mu} \frac{\partial \theta}{\partial x} + \frac{\sigma_1 E_y B_z - \sigma_0 E_z B_y}{\mu} - \frac{mN_0(1 + \eta)}{\mu} g$$

$$\alpha_1 = \frac{-a\omega + \sqrt{a^2\omega^2 + 4b(k^2 + p^2)}}{2(k^2 + p^2)} \quad \text{and} \quad \alpha_2 = \frac{-a\omega - \sqrt{a^2\omega^2 + 4b(k^2 + p^2)}}{2(k^2 + p^2)}$$

are the roots of auxiliary equation.

The expressions of Joule heating (Q_J) and viscous heating (Q_V) can be deduced from:

$$Q_J = \sigma_1 E_x^2 + \sigma_1 E_y^2 + \sigma_0 E_z^2 + \sigma_1 (B_z E_x - B_x E_z)v$$

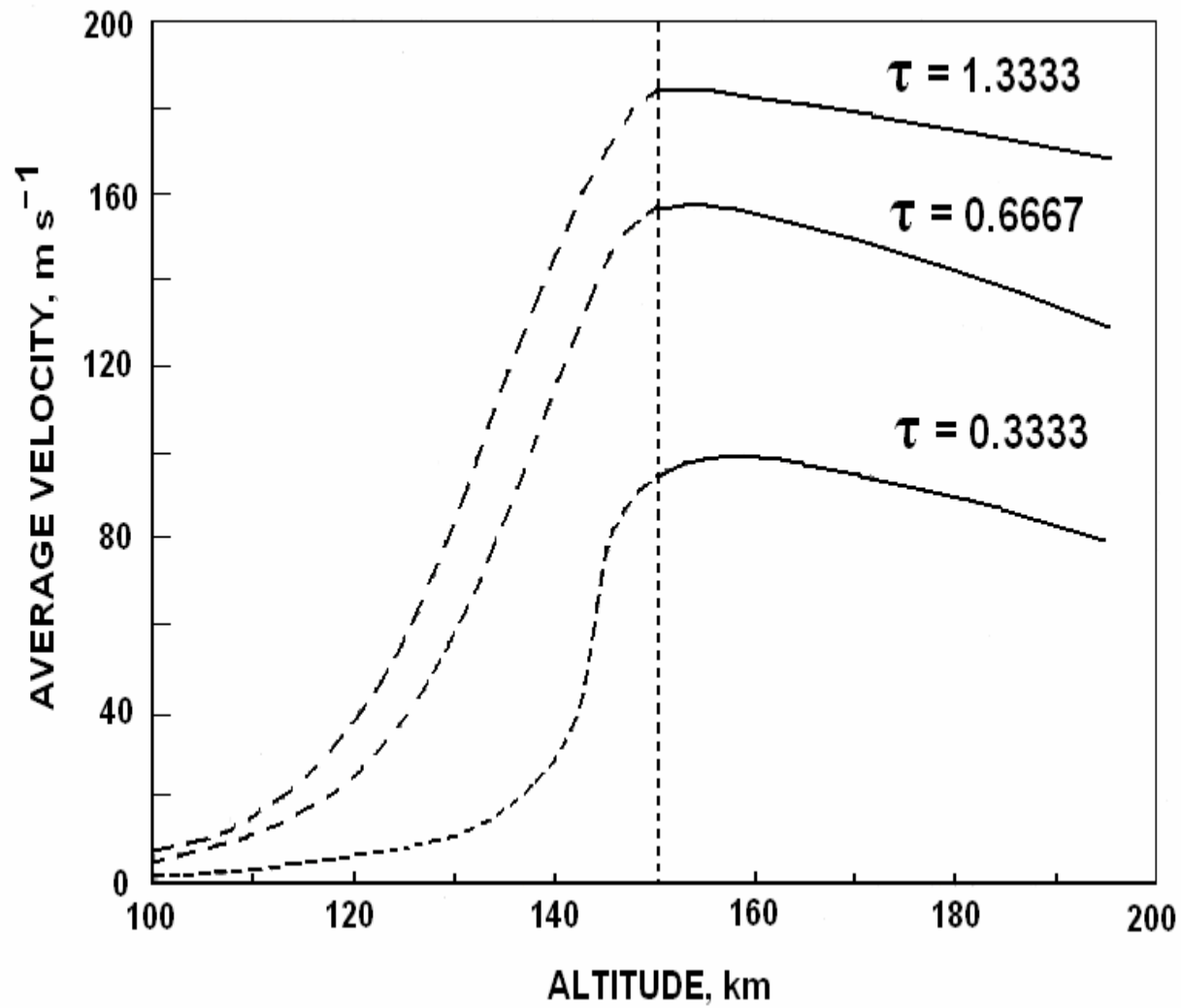
$$Q_V = \mu\nu(k^2 + p^2) \{A\alpha_1^2 \exp[\alpha_1(kx + pz - \omega t)] + B\alpha_2^2 \exp[\alpha_2(kx + pz - \omega t)]\}$$

The ratio of viscous heating rate to Joule heating rate terms can be readily obtained explicitly from the above two expressions as:

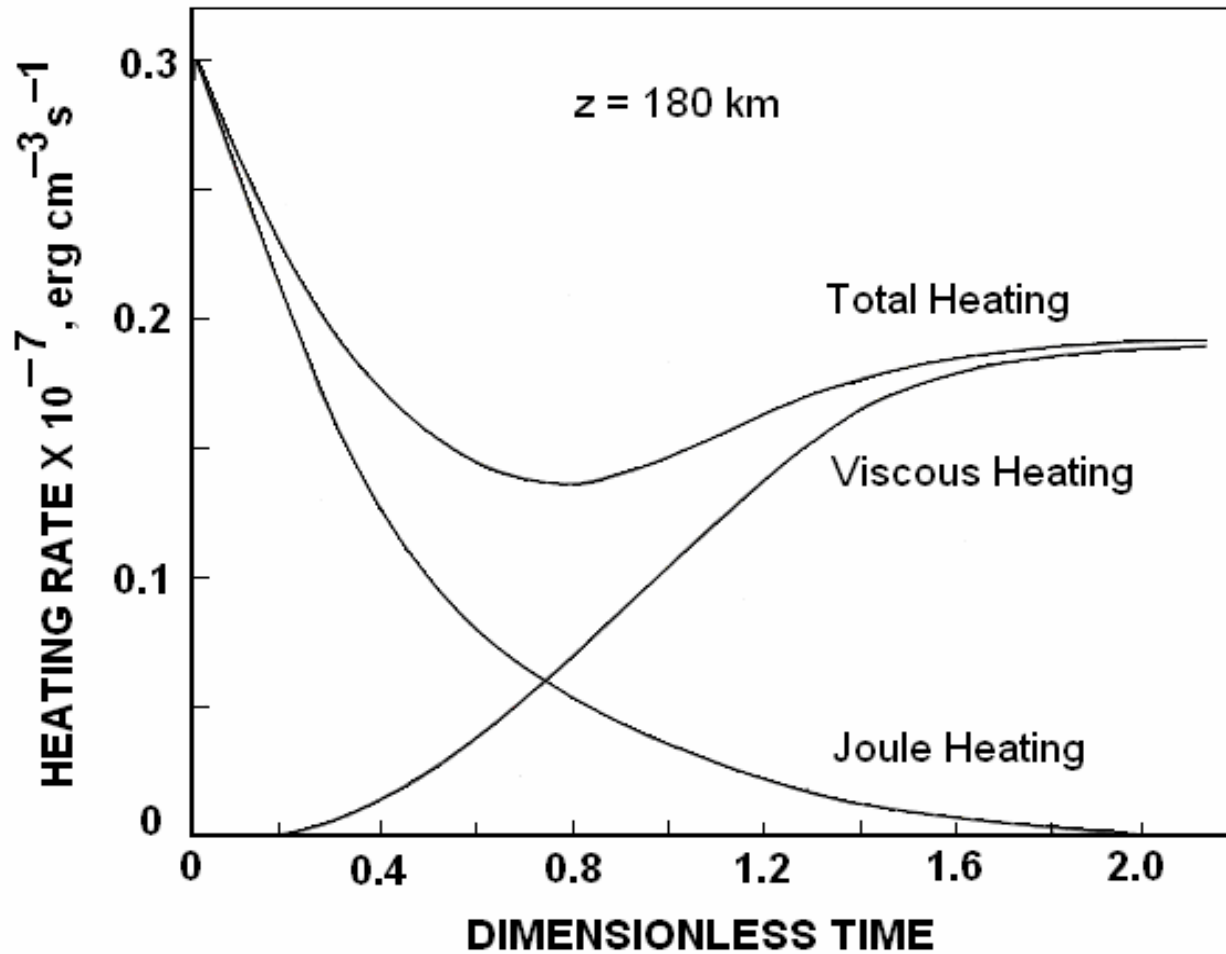
$$\begin{aligned}
& \mu(k^2 + p^2) \{ A \alpha_1^2 \exp[\alpha_1(kx + pz - \omega t)] + \{ B \alpha_2^2 \exp[\alpha_2(kx + pz - \omega t)] \} \times \\
& \times \{ A \exp[\alpha_1(kx + pz - \omega t)] + \\
& + B \exp[\alpha_2(kx + pz - \omega t)] - \frac{[k_B N_0 T_0 \frac{\partial \theta}{\partial x} + \sigma_1 E_y B_z - \sigma_0 E_z B_y - m N_0 (1 + \eta) g]}{m \nu_e N_0 (1 + \eta) + \sigma_1 B_x B_y} \} \\
R = & \frac{\sigma_1 E_x^2 + \sigma_1 E_y^2 + \sigma_0 E_z^2 + \sigma_1 (B_z E_x - B_x E_z) \times \{ A \exp[\alpha_1(kx + pz - \omega t)] + \\
& + B \exp[\alpha_2(kx + pz - \omega t)] - \frac{[k_B N_0 T_0 \frac{\partial \theta}{\partial x} + \sigma_1 E_y B_z - \sigma_0 E_z B_y - m N_0 (1 + \eta) g]}{m \nu_e N_0 (1 + \eta) + \sigma_1 B_x B_y} \} }{
\end{aligned}$$

The time constant required to reach the steady state, the characteristic time:

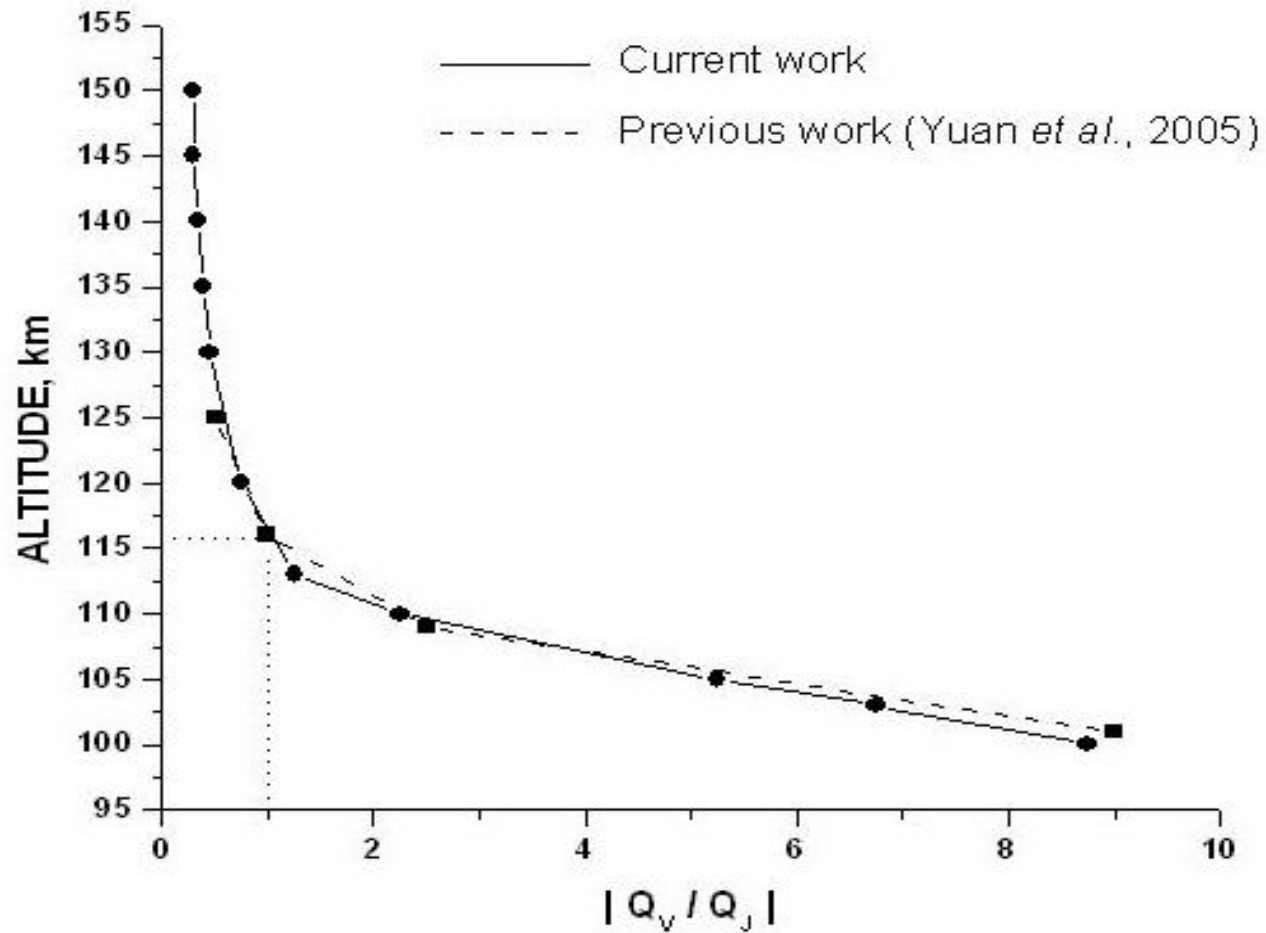
$$\tau = \frac{mN}{\sigma_1 B^2}$$



Average velocity vs. altitude at different characteristic times



Heating rate vs. dimensionless time at $z=180$ km



Variation of the ratio of viscous heating rate to Joule heating rate with altitude

CONCLUDING REMARKS

- ❑ Auroral heating of the neutral atmosphere is a part of the more general question of the total energy budget. The propagation of gravity waves is generated mainly in the E-region of the ionosphere by Joule heating and Lorentz forcing.
- ❑ Joule dissipation is a major ionospheric energy source at auroral latitudes that seems to be several times greater than that directly associated with particle precipitation.
- ❑ Joule heating is effective in the initial stage which decreases as the volume increases with time. When the motion is set-up due to dominating electric field, then only viscous heating results. This is increased due to movement and eventually it reaches a steady value.
- ❑ Between 100 and 125 km altitude, the ratio viscous heating rate to Joule heating rate ranges from 0.5 to 10 while between 125 and 150 km, it is approximately constant and the value obtained is 0.3. Under the height about 116 km the Lorentz force is relatively important and Joule heating dominates above the height.

MODE COUPLING

The secondary waves due to the coherent interaction of electromagnetic waves within the ionosphere have significant utility in the fields of radio-communication over long distances.

Such interaction within the ionosphere is considered.

The regions of possible coupling between ordinary and extraordinary modes during transverse wave propagation within the medium leading to the generation of ordinary mode will be presented.

The expressions of current and field during interactions are derived.

In the presence of irregularities, the fluctuations are introduced by the method of stretching. The second-order equations may be written as:

$$\left. \begin{aligned} \nabla \times \vec{H}_2 &= \frac{1}{c} \frac{\partial}{\partial t} [(\varepsilon_0) \vec{E}_2 + (\varepsilon_1) \vec{E}_1] \\ \nabla \times \vec{E}_2 &= \frac{i\omega}{c} \vec{H}_2 \end{aligned} \right\}$$

From these, one can get

$$[\nabla \nabla \cdot - \nabla^2 - \frac{\omega^2}{c^2} (\varepsilon_0)] \vec{E}_2 = \frac{2\omega^2}{c^2} (\varepsilon_1) \vec{E}_1$$

Where, (ε_0) and (ε_1) are the zeroth order and first order matrices of (ε) derived from the appropriate form of Lorentz force equation consistent with the nature of the medium.

$$(\varepsilon) = I + \frac{i\omega_p^2}{\omega\chi(\eta - i2\omega)} \times \begin{pmatrix} \chi_x & \Omega_x\Omega_y + \Omega_z(\eta - i2\omega) & \Omega_z\Omega_x - \Omega_y(\eta - i2\omega) \\ \Omega_x\Omega_y - \Omega_z(\eta - i2\omega) & \chi_y & \Omega_y\Omega_z + \Omega_x(\eta - i2\omega) \\ \Omega_z\Omega_x + \Omega_y(\eta - i2\omega) & \Omega_z\Omega_y - \Omega_x(\eta - i2\omega) & \chi_z \end{pmatrix}$$

Where $\Omega = \frac{eH}{mc}$

$$\omega_p^2 = \frac{4\pi ne^2}{m}$$

$$\chi = (\eta - i2\omega)^2 + \Omega^2$$

For ordinary and extra-ordinary waves, it is considered:

$$E_1^{(0)} = \hat{z}\xi_{\parallel}e^{i\varphi} \quad H_1^{(0)} = \hat{x}k_y \frac{c}{\omega} \xi_{\parallel}e^{i\varphi}$$

$$E_1^{(e)} = A(a\hat{x} + \hat{y})e^{i(\vec{k}\cdot\vec{r} - \omega t)} = \xi_e e^{i(kz - \omega t)}$$

$$H_1^{(e)} = -\hat{z}k_y \cdot \frac{cAa}{\omega} e^{i\varphi}$$

Where

$$k = \hat{y}k_y + \hat{z}k_z \quad \varphi = k_y y - \omega t$$

$$A^2 = \frac{|\xi_e|^2}{|a|^2 + 1} \quad a = \frac{i\omega_{p0}^2 \Omega_{0z}}{\omega\chi_0 + i\omega_{p0}^2(\eta - i2\omega)}$$

A= the normalization factor.

$$\omega_{p0}^2 = \frac{4\pi n_0 e^2}{m} \quad \Omega_{0z} = \frac{eH_{0z}}{mc}$$

$$\chi_0 = (\eta - i2\omega)^2 + \Omega_0^2$$

The process of coupling can be obtained through the part

$$\text{Re} \frac{2\omega^2}{c^2} (\varepsilon_1) \vec{E}_1 = \vec{J}_s$$

During interaction between ordinary (ω_1) and extra-ordinary (ω_2) modes of propagation,

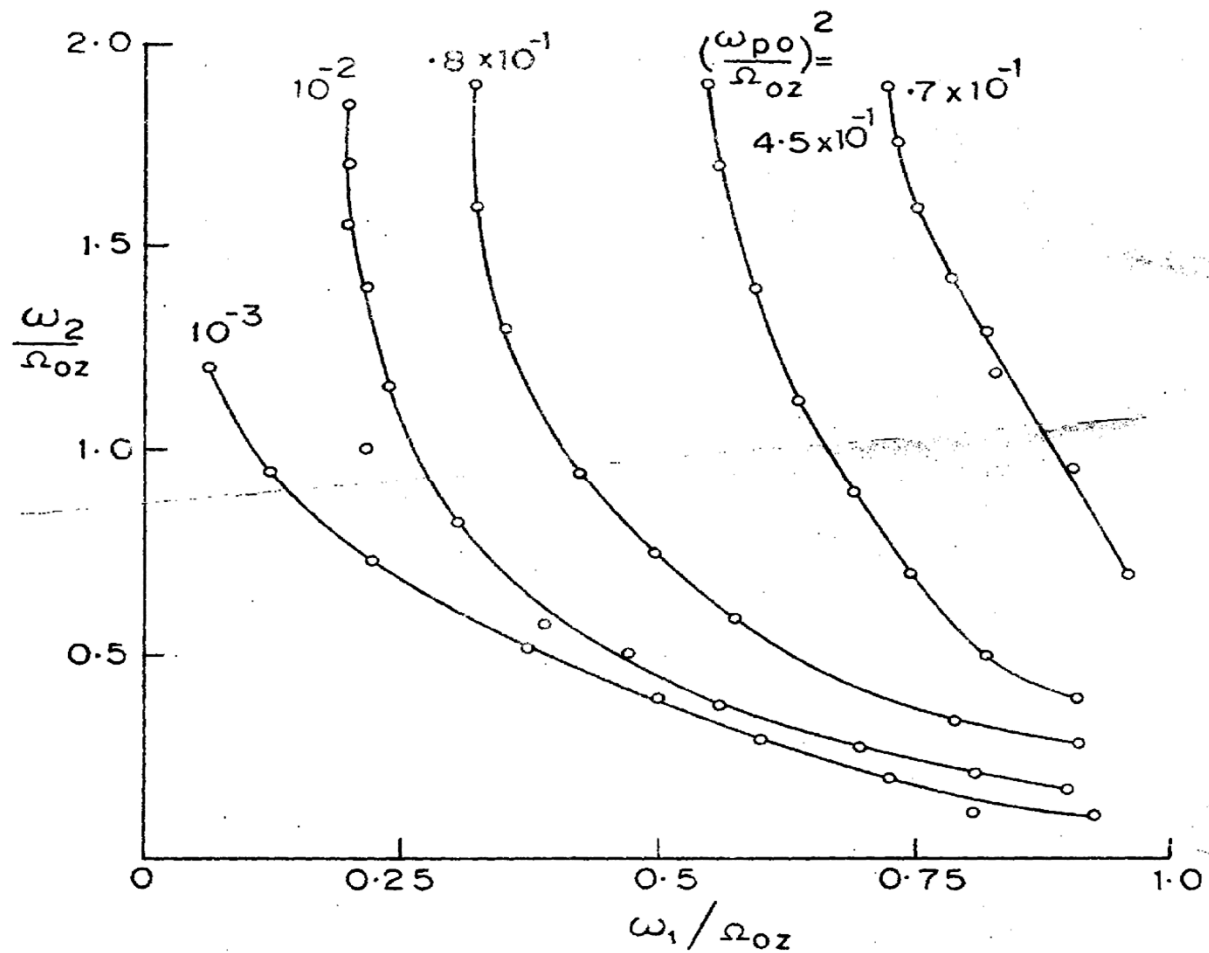
the expression of second-order current is obtained as

$$\vec{J}_S = \hat{z} \frac{\omega_{\pm}^2}{c^2} A e^{-(K_1'' + k_2'')y} [\{-\xi_1 a'(\zeta_1 k_2'' + \zeta_2 k_2') \mp \xi_1 a''(\zeta_1 k_2' - \zeta_2 k_2'') + \xi_2(\zeta_3 k_2'' + \zeta_4 k_1')\} \cos(k_{\pm}y - \omega_{\pm}t) + \{\mp \xi_1 a'(\zeta_1 k_2' - \zeta_2 k_2'') + \xi_1 a_2'(\zeta_1 k_2'' + \zeta_2 k_2') \pm \xi_2(\zeta_3 k_2' - \zeta_4 k_2'')\} \sin(k_{\pm}y - \omega_{\pm}t)]$$

and the expression of electric field is obtained as

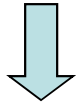
$$\vec{E}_2 = \vec{J}_S [k_{\pm}^2 - \frac{\omega_{\pm}^2}{c^2} (1 - \frac{2\omega_{P0}^2}{\eta^2 + 4\omega^2})]^{-1}$$

where symbols have their own meaning.

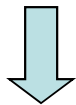


Interaction of ordinary wave (ω_1) with extraordinary wave (ω_2)
 leading to the generation of ordinary wave ($\omega_1 - \omega_2$)

For different value of plasma frequency (ω_p), the value of ω_1 and ω_2 are varied in step.



There is a curve in the ($\omega_1 - \omega_2$) plane which characterizes the allowable regions of interaction between ordinary and extra-ordinary modes of propagation.



The regions of possible coupling between the modes that lead to the generation of ordinary mode $\omega_1 \pm \omega_2$, have been shown.

The coupling between ordinary and extra-ordinary modes can be important under quasi-transverse condition.

Coupling phenomena at various heights of the ionospheric medium have been investigated by different investigators. EISCAT observations are being used successfully to investigate the dynamical coupling of the auroral F-region ionosphere and thermosphere (Gerrard *et al.*, 2004; Forbes and Harel, 1989).

High power and high frequency heating experiments from different locations of the world have been used to explore the electrical processes of the lower and upper atmosphere along with various possible sources of energy transfer between the near and far above the earth surface. Transport of electromagnetic energy from the atmosphere to the magnetosphere via ionosphere and back to the earth surface via the ionosphere and lower atmosphere through various coupling mechanism is a subject of current interest (Siingh *et al.*, 2005; Pulkkinen *et al.*, 2007).

Through the present analysis, an attempt has been taken which is enable to study a physical process within the disturbed ionosphere.

Thank You