

# Studies on non-linear heating in the polar ionosphere

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**Abstract**— The mechanisms for non-linear heating in the polar ionosphere due to auroral currents have been explored under time varying situation. The expressions of the Joule heating rate and the frictional heating rate due to interactions between neutrals and ions are deduced. The fluctuating electric field in connection with the auroral electrojet enhances the magnitude of frictional heating significantly in the polar ionosphere. The effective recombination coefficient in polar E-region is found to be reduced to a considerable extent due to high electric fields [1]. This is a consequence of high electron temperature and this has been incorporated in the present derivation. The height dependent rate of frictional heating for the ions due to collisions with the neutrals can be numerically estimated in presence of oscillatory auroral electric field.

**Keywords**— Non-linear heating, auroral currents.

## I. INTRODUCTION

In the polar ionosphere, particularly, the auroral E-region, there are always non-linear Joule heating and frictional heating due to interactions between ions and neutrals. The heating rates vary depending on the presence of oscillating electric fields which are likely to occur in the auroral E-region. This field represents a heat source in the medium.

When electric fields are fluctuating, the frictional heating rate is greatly enhanced for frequencies surrounding E-region ion gyrofrequencies [2]. Thus, in the total energy budget of the high latitude atmosphere, this additional heat source must have important effects along with Joule heating derived from ionospheric electric currents and fields.

In this presentation, the modified expressions of frictional heating rate is deduced theoretically for the stated physical situation taking into account the contributions of other dissipative processes involved in the phenomena.

In the presence of high electric fields, the effective

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recombination coefficient in the polar E-region reduces substantially. It is found that, the total ion production rate and particle heating are directly affected by the reduced recombination coefficient. The height dependence rate of frictional heating for the ions due to collisions with the neutrals due to an oscillating electric field can be studied numerically.

## II. FORMULATION

Ionospheric E-region plasma can be described by the quasi-hydrodynamic equations. The plasma consists of electrons, ions and neutrals. The equation of motion governing the plasma flow can be written as

$$\rho \left[ \frac{\partial v'}{\partial t} + v' \frac{\partial v'}{\partial x} \right] + \frac{\partial p}{\partial x} - \nu_m v' - \frac{4}{3} \mu \frac{\partial^2 v'}{\partial x^2} = \sigma (EB + v' B_0^2) \quad (1)$$

The continuity equations for positive and negative ions are

$$\frac{dn_j^+}{dt} = q_j + k_1 n_k^+ - k_2 n_j^+ - \sum_k \alpha_i(jk) n_j^+ n_k^- - \alpha_D(j) n_j^+ N_e \quad (2)$$

$$\frac{dn_j^-}{dt} = \beta N_e + k_1 n_k^- - k_2 n_j^- - \gamma_j n_j^- - \sum_k \alpha_i(jk) n_j^- n_k^+ \quad (3)$$

The simplified form of eqs.(2) and (3) can be written as

$$\frac{dN^+}{dt} = q - \alpha_i N^+ N^- - \alpha_D N^+ N_e \quad (4)$$

$$\frac{dN^-}{dt} = \beta N_e - \gamma N^- - \alpha_i N^- N^+ \quad (5)$$

where,  $\rho$  = mass density;

$$\frac{\partial p}{\partial x} = c_1 v';$$

$$c_1 = v'B_z \left[ 1 + \frac{1}{2} \left( \frac{\omega\rho}{B_z} \right)^2 \right]; \mu = \text{molecular viscosity,}$$

$B_z$  = magnetic field in the z-direction;  $B$  = geomagnetic field;  $\vec{v}' = \vec{v} - \vec{u}$ ;  $\vec{v}$  = ion velocity;  $\vec{u}$  = neutral particle velocity;  $\sigma$  = Pedersen conductivity given by [3] as

$$\sigma = \frac{N_0 e}{2m\omega_c} \left[ \sqrt{\frac{\pi}{2}} \frac{\omega_c}{2k_z v_0} e^{-\frac{k_z^2 v_0^2}{2\omega_c^2}} + \frac{\omega_c(\omega - \zeta)}{k_z^2 v_0^2} \right] \quad (6)$$

$\zeta = \nu + \nu$ ;  $\omega_c = \frac{eH}{m}$ ;  $\nu$  = ion-neutral collision frequency;  $\nu$  = electron-neutral collision frequency and other symbols are given Ref. [3]

$q_j$  = ion-production rate;  $\alpha_i(jk)n_j^+n_k^-$  = ion-ion recombination terms;  $\alpha_D(j)n_j^+N_e$  = dissociative recombination terms;  $\beta N_e$  = attachment rate;  $\alpha_i(jk)n_j^-n_k^+$  = ion-ion recombination terms. The other symbols have their usual meanings.

By Laplace transformation method, the eq. (1) has been solved for  $v'$  as

$$\begin{aligned} v'(x,t) = & D_1 \cosh(\sqrt{k_1}x) - D_2 \cosh(d_1x) \pm \\ & \pm D_3 e^{\beta t} \cos \beta_n x + (D_4 + \chi_1) \sinh(\sqrt{k_3}x) - \\ & - D_5 e^{\beta t} \sinh(2\sqrt{k_1}x) \mp U_1 e^{-\beta_2 t} \sin(\beta'_n x) + \dots \\ & + 17 \text{ terms,} \end{aligned} \quad (7)$$

containing elaborate substitutions.

The frictional heating rate can now be expressed as

$$\begin{aligned} \dot{Q} = & \frac{m_n}{m_n + m_i} \{ \sigma E_0^2 \sin^2 \omega t + \sigma B^2 \left( \frac{1}{\Omega_i} \frac{dv'}{dt} \right)^2 - \\ & - \frac{2\sigma B \alpha_i}{\Omega_i} \frac{dv'}{dt} E_0 \sin^2 \omega t - \left( \frac{q}{\alpha_D} \right) - \beta N^+ - \\ & - \frac{\gamma N^-}{N_e} \cos^2 \omega t \end{aligned} \quad (8)$$

The oscillatory electric field has been taken as  $E' = E_0 \sin \omega t$ , where  $\vec{E}' = \vec{E} + \vec{u} \times \vec{B}$ ;

$\Omega_i = \frac{eB}{m_i}$ . The first term of eq. (8) yields Joule heating

while the rest are related to inertia of ions, influence of ionization and recombination processes within the plasma.

### III. DISCUSSIONS

The total ion production rate and neutral particle heating are expected to be influenced by the reduced magnitude of the recombination coefficient. The height dependence of frictional heating for the ions due to collisions with the neutral particles can be numerically estimated from this work in presence of oscillatory auroral electric field. The result may justify the existence of considerable power within the E-region height range of the ionosphere obtained by rocket observations of electric field fluctuations in connection with an auroral electrojet [4].

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